Exponentially Large Extra Dimensions

A. Albrecht,[a] C.P. Burgess,[b,c] F. Ravndal [d] and C. Skordis [a]

[a] *Department of Physics, UC Davis, 1 Shields Avenue, Davis CA, USA 95616.*

[b] *School of Natural Sciences, Institute for Advanced Study, Princeton NJ, USA 08540.*

[c] *Physics Department, McGill University, 3600 University Street, Montréal, Québec, Canada H3A 2T8.*

[d] *Institute of Physics, University of Oslo, N-0316 Oslo, Norway.*

We show how the presence of a very light scalar with a cubic self-interaction in six dimensions can stabilize the extra dimensions at radii which are naturally exponentially large, $r \sim \ell \exp[+(4\pi)^3/g^2]$, where ℓ is a microscopic physics scale and g is the (dimensionless) cubic coupling constant. The resulting radion mode of the metric becomes a very light degree of freedom whose mass, $m \sim 1/(M_p r^2)$ is stable under radiative corrections. For $1/r \sim 10^{-3}$ eV the radion is extremely light, $m \sim 10^{-33}$ eV. Its couplings cause important deviations from General Relativity in the very early universe, but naturally evolve to phenomenologically acceptable values at present. Such a radion provides a completely natural 'Planck-Scale Quintessence', providing an attractive description of the observed cosmological acceleration, with interactions which are consistent with experiments.

PACS numbers:

I. INTRODUCTION

Recent developments in both particle physics and cosmology appear to indicate the existence of important fundamental physics associated with the scale 10^{-3} eV.

- On the cosmological side, physics at this scale appears to be indicated by the recent observational evidence [1] for the existence of a 'dark energy' component to the universe (possibly a cosmological constant), having a negative pressure of order $p \sim (10^{-3}\,\mathrm{eV})^4$.

- On the particle physics side several developments have made plausible the existence of interesting 10^{-3} eV physics. On the one hand, it has long been recognized that breaking supersymmetry at the TeV scale would imply masses of order $m \sim (1\mathrm{TeV})^2/M_p \sim 10^{-3}$ eV for the gravitino, and other gravitationally-coupled particles. Here $M_p \sim 10^{18}$ GeV is the (rationalized) Planck mass.

- More recently has come the recognition that extra dimensions can be much larger (and have much richer dynamics) than had hitherto been appreciated, and the realization that such large extra dimensions could help solve some long-standing problems like the heirarchy problem [2,3]. In particular, these dimensions could have radii as large as $r \sim (10^{-3}\mathrm{eV})^{-1}$.

Interest in all of these issues has been sharpened by the realization that the existence of new particles at this scale may have other observational consequences. The resulting modifications to Newton's Law of Gravity may fall within reach of current and upcoming experiments. The dynamics of red giants and supernovae can be modified, and the resulting bounds can be important [4,5], but need not be fatal [6,7].

A crucial part of any large-extra-dimension scenario is a natural mechanism for generating a radius which is large compared to other microphysical scales. This is a particularly pointed requirement if the large ratio, $(10^{-3}\mathrm{eV})/(1\mathrm{TeV})$, is to be used to explain other heirarchies, like M_W/M_p.

It is our purpose in this paper to propose a mechanism for generating such large dimensions. Although our proposal is not restricted to radii as large as $r \sim (10^{-3}\mathrm{eV})^{-1}$, it can satisfy the very restrictive phenomenological constraints which apply in this case. Furthermore, we argue that the energetics which chooses the value for the extra-dimensional radius in our mechanism can have attractive cosmological consequences when the extra dimensions are large.

Our proposal is based on the observation that large radii are naturally obtainable if the potential which governs the radion is logarithmic:*

$$\ell^4 V(r) = \left(\frac{\ell}{r}\right)^p \left[a_0 + a_1 \log\left(\frac{r}{\ell}\right) + \cdots\right] + O\left[\left(\frac{\ell}{r}\right)^q\right], \tag{1}$$

(where $q > p$ and ℓ is a microscopic length scale). Besides the usual runaway solution, $r \to \infty$, the stationary condition $dV/dr = 0$ also admits the solutions:

$$\frac{r}{\ell} \approx \exp\left(\frac{1}{p} - \frac{a_0}{a_1}\right), \tag{2}$$

*For another approach to obtaining logarithmic radion potentials in six dimensions, see [7].

where the approximation becomes exact in the absence of higher powers of $\log(r/\ell)$. This basic mechanism was proposed at a phenomenological level in the "Planck scale quintessence" models [8]. In that context it was shown that the time for quantum tunneling out of the local minimum was exponentially larger than the age of the Universe [9].

Eq. (2) predicts r is naturally exponentially large compared to ℓ, if two conditions are satisfied:

1. a_0 and a_1 must have opposite signs; and
2. there is a modest heirarchy in the coefficients, a_k.

For instance if $a_1 = -\hat{a}_1\epsilon$, with $\epsilon < 1$ and a_0 and $\hat{a}_1$ both positive and $O(1)$, then $r/\ell = O(\exp[a_0/(\hat{a}_1\epsilon)+O(1)]) \gg 1$. Numerically, if $1/\ell \sim 1$ TeV and $a_0/(\hat{a}_1\epsilon) \sim 35$ then $1/r \sim 10^{-3}$ eV falls into the interesting range.

We here propose a scenario which generates logarithmic radion potentials which very generically satisfy both of these two conditions. The scenario has the radion potential generated as the universe passes through a stage during which it is effectively six-dimensional, provided that there is at least one six-dimensional scalar field whose mass is of order $1/r$ and which has reasonably large, nonderivative, cubic self-interactions.

In six dimensions a cubic scalar self-interaction,

$$U_{\rm ren}(\phi) = \frac{g}{3!}\,\phi^3, \tag{3}$$

is the only local interaction which has a dimensionless coupling constant. Because of this, in perturbation theory its renormalization gives it a logarithmic, rather than a power, dependence on r. In renormalization-group terms it is the only six-dimensional interaction which is not irrelevant in the infrared.

The logarithmic dependence of the low-energy coupling, $g(r)$, provides a natural way of obtaining logarithmic potentials for r. In the absence of light-scalar loops, for large r the radion potential has the generic form

$$V(r) = \sum_k \frac{c_k}{r^k}. \tag{4}$$

This can arise in particular examples in many ways. For instance, it arises when evaluating the classical action as a function of radius if six-dimensional gravity or supergravity is compactified on a sphere. Alternatively, it could dominantly arise as a quantum Casimir energy, such as in a toroidal compactification [10].

Scalar radiative corrections (in six dimensions) to this potential correct the constants c_k:

$$c_k = \sum_l c_k^{(l)}\,\left[\frac{g^2(r)}{(4\pi)^3}\right]^l, \tag{5}$$

where $g(r) = g_0 + bg_0^3\log(r/\ell) + O(g_0^5)$.

Using eq. (5) in eq. (4) produces a logarithmic potential of the desired type, with several remarkable features:

- Eq. (5) automatically introduces the desired heirarchy (and so satisfies condition 2 above) by systematically suppressing higher powers of $\log r$ with the suppression factor $\epsilon \sim g^2/(4\pi)^3$.
- The relative sign of the coefficients, a_0 and a_1, of the first two terms in eq. (1) depends crucially on the sign of b, the one-loop renormalization-group coefficient for g. Furthermore, they have opposite signs, as required to generate a heirarchy (condition 1 above) provided that the coupling g is asymptotically free, as is a cubic scalar coupling in six dimensions (for which $b = +3/(256\pi^3)$).
- Because the minimization of $V(r)$ with respect to r naturally occurs when $\alpha(r) := g^2(r)/(4\pi)^3 = O(1)$, the precise value for $r = r_s$ at the minimum cannot be computed perturbatively in $\alpha(r)$. (An exception to this statement arises if the lowest-order coefficient, $c_k^{(l_0)}$, happens to be much smaller than the others, as happens in an example below.) The order of magnitude of the r so obtained is nonetheless reliably predicted to be exponentially large – $r_s/\ell = O[\exp(+1/\alpha_0)]$, where $\alpha_0 = \alpha(\ell)$ – given the logarithmic running of α.

We present our argument in more detail in the subsequent sections. First, the next section gives more explicit expressions for the radion potential in a model consisting only of the metric and a very light scalar field. Since our mechanism expresses the explanation for large dimensions in terms of a light scalar field, we then follow with a discussion of how natural it is to find scalars with the required properties. We conclude with a discussion of the cosmological implications of the logarithmic potential, and the bounds which these may impose on model building.

II. RADION POTENTIALS AND LIGHT SCALARS IN 6 DIMENSIONS

In this section we compute explicitly the radion potential produced by a light scalar field for a simple compactification. To this end consider a model consisting of a scalar field, ϕ, and the six-dimensional metric, $\mathcal{G}_{MN}$. We imagine this to be an effective six-dimensional theory obtained after integrating out all more massive degrees of freedom at scale ℓ. The leading terms in the derivative expansion for this lagrangian have the form:

$$\frac{\mathcal{L}}{\sqrt{\mathcal{G}}} = -\,\frac{1}{2\ell^4}\mathcal{R} - \frac{1}{2}\,\mathcal{G}^{MN}\,\partial_M\phi\partial_N\phi - U(\phi), \tag{6}$$

where $\mathcal{R}$ denotes the scalar curvature built from the six-dimensional metric.

We assume the scalar potential to have the form

$$U(\phi) = \frac{\mu^2}{2}\,\phi^2 + \frac{g_0}{3!}\,\phi^3 + \frac{\kappa\ell^2}{4!}\,\phi^4 + \cdots \tag{7}$$

with the microscopic scale ℓ setting the dimensions of all but two of the couplings in the scalar potential. The two exceptions are: (i) we assume there is no cosmological constant term in $U(\phi)$; and (ii) we assume the scalar mass is small: $\mu \lesssim 1/r \ll 1/\ell$. In this section we simply fine-tune the lagrangian to ensure these conditions are satified, but since our generation of the logarithmic potential is based on these choices, in the next section we address how difficult they are to arrange within models. Although we shall argue that assumption (ii) is simple to arrange in supersymmetric models, the tricky part is to have μ be as small as $O(1/r)$ without also finding the cubic term similarly suppressed, $g_0 \sim \ell^2/r^2$.

A. One Loop Casimir Energy

Before searching for logarithmic corrections due to the cubic scalar coupling, we must first compute the potential of the form eq. (4) which is to be corrected. This we compute semiclassically assuming the ground state geometry is flat: $M^6 = R^4 \times T^2$, where R^4 denotes flat Minkowski space and T^2 is a torus, both of whose radii we denote by r. (Although the torus has other moduli besides its radius, here we focus only on r.) By choosing $\mu^2 > 0$ we have arranged $U(\phi)$ to have a local minimum at $\phi = 0$. Since the quadratic term in ϕ is destabilized by the assumed cubic term, we must also assume the potential to be bounded below by other terms in $U(\phi)$, involving higher powers of ϕ, for example. The detailed form of these higher terms do not play a role in the discussion which follows.

Under these circumstances the radion potential first arises as a Casimir energy at one loop. The scalar contribution to this energy is computed in Appendix A, and is given by:

$$V_1(r) = -\,\frac{1}{2r^4}\int_0^\infty \frac{dx}{x^3}\; e^{-\beta x}\;\left[\vartheta_j(ix)\,\vartheta_k(ix) - \frac{1}{x}\right], \tag{8}$$

where $\beta = \mu^2 r^2/(4\pi)$ and $j,k = 2,3$ depending on the boundary condition which the scalar satisfies about the torus' two nontrivial cycles. The choice $j,k = 3$ corresponds to periodic boundary conditions about these cycles, while $j,k = 2$ corresponds to antiperiodic boundary conditions. The functions $\vartheta_j(\tau)$ are the usual Jacobi theta-functions [11].

As is easily verified, eq. (8) converges in both the ultraviolet and infrared, even if $\mu \to 0$. If $\mu r \gg 1$ then V_1 falls exponentially as $\mu r \to \infty$. If $\mu \to 0$, then the potential takes the form of a power of r: $V_1 = c_4^{(1)}/r^4$, as in eq. (4), with:

$$\begin{aligned} c_4^{(1)}(j,k) &= -\,\frac{1}{2}\int_0^\infty \frac{dx}{x^3}\left[\vartheta_j(ix)\,\vartheta_k(ix) - \frac{1}{x}\right], \\ &= \begin{cases} -0.1502 & \text{if } j=k=3 \\ +0.0141 & \text{if } j=2, k=3 \\ +0.1142 & \text{if } j=k=2 \end{cases} \end{aligned} \tag{9}$$

and we recall $j,k = 3$ corresponds to periodic (and $j,k = 2$ to antiperiodic) boundary conditions. For later use when discussing models and cosmology, we remark that $c_4^{(1)}$ is positive if ϕ is antiperiodic around either of the two nontrivial cycles.

To this should be added the contribution due to the graviton Casimir energy, as well as the Casimir energy due to any other six-dimensional particles. Because of the rapid falloff in the result as $\mu r \to \infty$ it is clear that only the contribution of those degrees of freedom for which $\mu r \lesssim 1$ is important for large r. We consider these contributions in more detail in following sections, where the structure of the entire model is considered in more detail.

B. Radiative Corrections

We next turn to the corrections to $V_1(r)$ which dominate for large r.

The first observation to be made is that only the ϕ^3 interaction of $U(\phi)$ can contribute to $V(r)$ unsuppressed by further powers of $1/r$. This is because all other coupling constants have dimension of a positive power of length, and so are perturbatively nonrenormalizable.

To see how this works consider the ϕ^4 term in $U(\phi)$, whose coupling in eq. (7) is denoted $\kappa\ell^2$. Imagine now scaling out powers of r to make all couplings dimensionless, so the potential $U(\phi)$ is written

$$U(\phi) = \frac{(\mu r)^2}{r^2}\,\phi^2 + \frac{g_0}{3!}\,\phi^3 + \left(\frac{\kappa\ell^2}{r^2}\right) r^2\phi^4 + \cdot \tag{10}$$

Once written this way it is clear that each factor of the coupling κ in $V(r)$ is accompanied by a power of ℓ^2/r^2, implying a contribution which is further suppressed by powers of $1/r$ compared to the uncorrected term.[†]

[†]Complications to this dimensional argument arise if there are power-law infrared divergences. These do not occur in six dimensions, but arise in some circumstances from four-dimensional massless scalar states once we compactify to four dimensions, in the presence of cubic scalar self-interactions (which are super-renormalizable in four dimensions). These issues can be addressed purely within the effective four-dimensional theory applicable at distances larger than r, however, and do not arise at all for scalars satisfying antiperiodic boundary conditions on the torus. (We thank S. Shenker for conversations on this point.)

For corrections to $V(r)$ which are *not* suppressed by more powers of $1/r$ we must consider graphs which involve only the dimensionless cubic scalar self-coupling g_0. The emergence of the logarithms can then be most easily seen in the limit as $\mu \to 0$, in which case it is revealed by a simple renormalization-group argument. For $\mu \to 0$ the dominant radion corrections can be written, on dimensional grounds, as:

$$V(r) = \frac{1}{r^4}\Big[A_0 + A_1(r/r_0)\alpha(r_0) + A_2(r/r_0)\alpha^2(r_0) + \cdots\Big], \tag{11}$$

where $\alpha = g^2/(4\pi)^3$ is the six-dimensional loop-counting parameter, $g(r_0)$ is the renormalized coupling, renormalized at an arbitrary renormalization point, r_0. The coefficients $A_k(r/r_0)$ can be dimensionless functions of r/r_0, although explicit calculation has just shown $A_0 = c_4^{(1)}$ to be a constant.

Since the dependence of V on r is tied to its dependence on r_0, and since V cannot depend on r_0 at all, the r dependence of the A_k's can be related to the running of α. That is, if:

$$r_0\frac{d\alpha}{dr_0} = B\alpha^2 + O(\alpha^3), \tag{12}$$

then the Callan-Symanzik equation, $r_0 dV/dr_0 = 0$, implies $dA_1/dr = 0$ and $r dA_2/dr = B\,A_1$. For renormalization schemes for which B is r-independent, this implies:

$$\begin{aligned} V(r) &= \frac{1}{r^4}\Big\{c_4^{(1)} + c_4^{(2)}\alpha(r_0) \qquad (13)\\ &\qquad + \alpha^2(r_0)\left[c_4^{(3)} + Bc_4^{(2)}\,\log\left(\frac{r}{r_0}\right)\right] + \cdots\Big\}\\ &= \frac{1}{r^4}\Big\{c_4^{(1)} + c_4^{(2)}\alpha(r) + c_4^{(3)}\alpha^2(r) + \cdots\Big\}, \end{aligned}$$

where $\alpha(r)$ is the solution to the one-loop renormalization flow $r d\alpha/dr = B\,\alpha^2$:

$$\alpha(r) = \frac{\alpha_0}{1 - B\,\alpha_0\,\log(r/\ell)}. \tag{14}$$

As usual, this expression is accurate to leading order in α_0, but to all orders in $\alpha_0\log(r/\ell)$.

Several conclusions may be drawn from eq. (13). First, it shows that the use of the renormalization group to resum all orders in $\alpha_0\log(r/\ell)$ permits the inference of the coefficient of every power of $\log(r/\ell)$ to leading order in $\alpha_0 = \alpha(\ell)$. Furthermore, eq. (13) establishes that this leading $\log(r)$ dependence is purely determined by the one-loop renormalization group coefficient, B, and the two-loop vacuum energy coefficient, $c_4^{(2)}$, whose evaluations are our next task.

Before turning to this task, there is another lesson to be drawn from eq. (13). This equation shows that the precise *value* of r which extremizes V cannot be computed in perturbation theory unless $c_4^{(1)}$ is much smaller than $c_4^{(2)}$. This is most easily seen in the second equality of eq. (13), for which the stationary point, r_s, is determined by the condition

$$-4c_4^{(1)} - 4c_4^{(2)}\alpha(r_s) + \alpha^2(r_s)[Bc_4^{(2)} - 4c_4^{(3)}] + \cdots = 0, \tag{15}$$

which is satisfied by $\alpha(r_s) = O(1)$ (unless $\left|c_4^{(1)}\right| \ll \left|c_4^{(2)}\right|$, in which case $\alpha(r_s) \approx -c_4^{(1)}/c_4^{(2)} \ll 1$). For our purposes, however, it is sufficient to know the order of magnitude of r_s, which *is* known perturbatively, and is reliably of order $\exp(+1/\alpha_0)$ compared to ℓ.

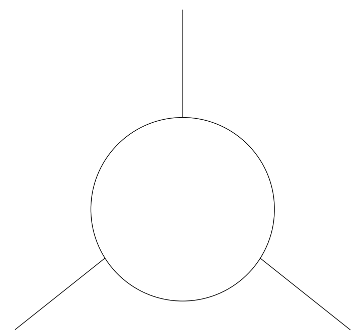

Figure 1: The Feynman graph which gives the one-loop counterterm for the cubic self-coupling, g.

Calculating B: The one-loop renormalization is computed by evaluating the graph of Fig. (1), with no momentum flowing through any of the external legs, plus the contribution due to the wavefunction renormalization of the scalar. Using a mass-independent renormalization scheme in six-dimensional (infinite-volume) Minkowski space these graphs imply a standard result: $B = +3/2$.

The calculation for a scalar field on the torus is straightforwardly performed using the techniques of Appendices A and B. If the renormalized value of g is defined as the third derivative of the effective potential evaluated at $\phi = 0$, then we find the following result for the counterterm for g:

$$\begin{aligned} \delta_{ct}g(r) &= \frac{g^3}{2(4\pi)^3}\int_0^\infty \frac{ds\,dt}{(s+t)^3}\;e^{-\beta(s+t)} \qquad (16)\\ &\qquad \times\left[s^2 + t^2 + \frac{1}{2}\,st\right]\vartheta_j[i(s+t)]\vartheta_k[i(s+t)], \end{aligned}$$

where, as before, $\beta = \mu^2 r^2/(4\pi)$.

This expression diverges logarithmically in the ultraviolet with a coefficient which reproduces the well-known continuum result. For antiperiodic boundary conditions, eq. (16) is infrared finite, and so in this case we may take

$\mu \to 0$. On dimensional grounds r can then only appear with the ultraviolet regulator scale, again implying the continuum result $B = +3/2$. For periodic scalars $\delta_{ct} g$ diverges in the infrared like $1/\mu^2$ as $\mu \to 0$. Although this properly describes the crossover in this case to the infrared behaviour of a massless scalar in four dimensions, it complicates the expression for B. In what follows we focus on the antiperiodic case.

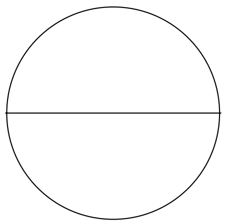

Figure 2: The Feynman graph which gives the two-loop contributions to the Casimir energy. To this must be added the one-loop contribution into which the one-loop self-energy counterterm is inserted.

Calculating $c_4^{(2)}$: The two-loop correction to the vacuum energy arises from the two-loop vacuum graph of Fig. (2). As computed in detail in Appendix B, after renormalization this gives the following finite contribution to the radion potential:

$$\begin{aligned} V_2(r) &= \frac{\alpha_0}{r^4}\, f_{jk}(\mu r), \\ &\to c_4^{(2)}(j,k)\, \frac{\alpha_0}{r^4} \qquad (\mu \to 0), \end{aligned} \tag{17}$$

where the function $f_{jk}(\mu r)$ is computed in Appendix B, and in the second line the limit $\mu \to 0$ has been taken. Numerical integration of the expressions given in Appendix B gives the constant $c_4^{(2)}(j,k) = f_{jk}(0)$ to be

$$c_4^{(2)}(j,k) = \begin{cases} -0.17 & \text{if } j = k = 3 \\ -0.10 & \text{if } j = 2, k = 3 \\ -0.09 & \text{if } j = k = 2 \end{cases} \tag{18}$$

Recall that it is the boundary conditions $(j,k) = (2,3)$ and $(2,2)$ which most interest us if $V(r_s)$ is to represent the current cosmological constant, because it is only for these choices that $V(r_s)$ is positive. Notice that for both of these choices $c_4^{(2)}(j,k)$ has the opposite sign from $c_4^{(1)}(j,k)$, a fact which is crucial for the existence of a nontrivial large extremum for $V(r)$ at large r.

Notice also that for $(j,k) = (2,3)$ – and only for this choice – we have $\left| c_4^{(1)}/c_4^{(2)} \right| = 0.14$, a value which allows a perturbative understanding of the properties at the minimum, since

$$\alpha(r_s) = -c_4^{(1)}/c_4^{2)} = +0.1 + O[\alpha^2(r_s)]. \tag{19}$$

To summarize, we see that a very light six-dimensional scalar field with a cubic self-coupling term can naturally generate logarithmic radion potentials. For instance, in the case of a massless scalar ($\mu = 0$) compactified on a torus, and satisfying antiperiodic boundary conditions about one or more of the torus' cycles, the leading large-r potential which is generated is given by eqs. (13) and (14), with $B = +3/2$ and $c_4^{(1)}$ and $c_4^{(2)}$ are given by eqs. (9) and (18). Notice that the leading coefficients of all powers of $\log r$ are determined by the running of the coupling α, and are resummed by standard renormalization group arguments.

Furthermore, this logarithmic potential naturally has the features identified in the introduction to generate an exponentially large heirarchy for r/ℓ, since higher powers of logarithms have systematically smaller coefficients. This is all the more remarkable given that the only choices which can be made are the boundary conditions which are satisfied by the scalar. In particular, the relative signs required for the existence of exponentially large stationary points for $V(r)$ need not have worked out as well as they did.

To this point, however, we are trading the puzzle of why r/ℓ is large for the puzzle of why there should be a light six-dimensional scalar field with cubic self-couplings. We now turn to a search for microscopic models which might be expected to have such scalars.

III. TOWARDS MICROSCOPIC MODEL BUILDING

In this section we ask what is required of a more microscopic model in order to provide the desired light scalar, whose couplings generate the logarithmic potential. Our purpose is twofold. We first intend to describe general features any such model must have, and to identify the naturalness issues which any such model must address. Although we examine several six-dimensional supergravity models in detail, we do not succeed in obtaining a completely natural candidate.

A. Model-Building Issues

The framework for more microscopic model-building depends crucially on how large are the radii which we are willing to contemplate. The two main options divide over whether r is larger or smaller than $(1\,\mathrm{TeV})^{-1}$. If $r \leq (1\,\mathrm{TeV})^{-1}$, considerable latitude exists because there are fewer constraints on the model-building. The intermediate-scale string [12] provides an attractive version of this scenario with $1/\ell \sim 10^{10}$ GeV, $1/r \sim 1$ TeV and $1/(M_p r^2) \sim 10^{-3}$ eV. Here we will focus on the more ambitious option, with $r \gg (1\,\mathrm{TeV})^{-1}$, with an eye to applications for which $1/r \sim 10^{-3}$ eV. For radii this large, some general observations are immediate.

For radii larger than the weak scale, model-building must take place within the brane-world scenario, in which all (or most) standard-model particles are confined to a four-dimensional surface within the larger six-dimensional space [2,3]. (There may also be other particles confined to other branes, with which ordinary particles only couple indirectly, the the exchange of 'bulk' states, which are free to move throughout the six dimensions. There is some freedom to choose what kinds of particles live in the bulk, but the bulk sector must include the graviton. In what follows we assume this framework to be true, although for simplicity, and in keeping with our earlier calculations with flat space, we imagine no cosmological constant in the six dimensions (the ADD scenario [2]), with gravitons not localized around the brane.

In this case $1/r \sim 10^{-3}$ eV is as large a radius as can be contemplated, due partly to the many bounds on modifications to Newton's Law on scales larger than a millimetre [13], and partly to limits on particle emission in astrophysical environments like supernovae [4,5]. We believe models predicting radii close to 10^{-3} eV are not yet ruled out in principle by these bounds, although they must be rechecked once specific models are proposed.

In any braneworld scenario with $1/r \sim 10^{-3}$ eV, there can be at most six dimensions, and the scale of physics on the branes themselves must be $M_b \sim 1$ TeV. This is because the four-dimensional Planck's constant in these models is of order $M_p^2 \sim M_b^{2+n} r^n$, where n is the number of extra dimensions. For $n = 2$ (six dimensions) this states that $M_b \sim (M_p/r)^{\frac{1}{2}} \sim 1$ TeV, as claimed. For $n > 2$, M_b is unacceptably low: *e.g.* when $n = 3$, $M_b \sim (M_p^2/r^3)^{\frac{1}{5}} \sim 1$ GeV. (Of course it was this argument run backwards which led early workers to contemplate radii as large as we are considering.)

We seek models having very light, six-dimensional scalars whose cubic self-couplings are not systematically suppressed by powers of $1/r$. The first of these conditions is actually very easy to achieve, since the existence of very light six-dimensional scalars arises very naturally within the braneworld framework. To be six-dimensional the scalars must live in the bulk, and not be localized on the brane (as are ordinary particles, like photons and electrons). Such scalars would be sufficiently light if they were tied by a symmetry to another massless bulk particle, such as the graviton.

For instance, in any supersymmetric variant of the braneworld scenario (with millimetre-scale extra dimensions) supersymmetry must be directly broken at the brane scale, $M_b \sim 1$ TeV, and so cannot be hidden too much from ordinary matter in order to split supermultiplets by TeV scales. To the extent that bulk states only couple to the branes with gravitational strength – certainly true for the graviton supermultiplet – it follows that the mass splittings within the supermultiplets in the bulk are very small. They are small because they are suppressed by their weak gravitational couplings to the supersymmetry-breaking sector, and so are of order $\Delta m \sim M_b^2/M_p$. Indeed, if $M_b \sim 1$ TeV this mass splitting is precisely the size of interest to us: $\Delta m \sim 10^{-3}$ eV.

In this way we see that bulk scalars in general, and in particular scalars that are tied to the graviton by supersymmetry, can be naturally expected to have masses which are sufficiently small. They are sufficiently light because the connection between M_p and r in six-dimensional models ensures the coincidence of the scales $1/r$ and M_b/M_p^2. Furthermore, the gravitational strength of their couplings would have hidden them from discovery before now.

We are led to ask: do scalars arise in plausible representations of six-dimensional supergravity? The answer is 'yes', including (in a particular sense, explained next) the graviton multiplet itself. Although strictly speaking the basic six-dimensional graviton supermultiplet consists of the sechsbein, gravitino and a Kalb-Ramond field [14–16]:

$$\Big\{ e^A{}_M, \psi_M, B^+_{MN} \Big\}, \tag{20}$$

this multiplet does not admit a Lorentz-invariant action because the Kalb-Ramond field strength is self-dual. Consequently, six-dimensional supergravity lagrangians are written with the supergravity multiplet coupled to a Kalb-Ramond matter multiplet,

$$\Big\{ B^-_{MN}, \chi, \sigma \Big\}, \tag{21}$$

which contains an anti-selfdual Kalb-Ramond field plus a fermion and scalar. Thus the *nonchiral* gravity multiplet naturally contains a scalar, σ, which plays the role of the dilaton in low-energy string compactifications.

Other spinless particles, which are tied by supersymmetry to other massless states in the bulk – like gauge bosons or chiral fermions – are also candidates for our light scalar, although they introduce somewhat more model-dependence than does the dilaton.

So far, so good: we have a light, six-dimensional scalar. The thorny issue for making models is obtaining a sizeable cubic self-interaction for these scalars. On one hand, the explicit calculation of the light-scalar couplings is difficult to perform, since both the light scalar and the radion often have no potential energy at all unless supersymmetry is spontaneously broken. Since our understanding of the nature of supersymmetry breaking in brane models is poor, it is difficult to definitively decide how big the cubic (and other) couplings might be in the low energy theory, once the brane physics is integrated out.

On the other hand, it is very natural to assume that any cubic couplings obtained after supersymmetry breaking should be of the same order of magnitude as are the scalar masses themselves. For instance, to the extent that the scalar potential for a light scalar, ϕ, is a supersymmetry-breaking effect, one might expect the entire potential to have the schematic form:

$$U(\phi) = \left(\frac{M_b}{M_p}\right)^2 \left[\frac{k_1 M_b^2}{2}\,\phi^2 + \frac{k_2}{3!}\,\phi^3 + \frac{k_4}{4!\,M_b^2}\,\phi^4 + \cdots\right], \tag{22}$$

which expresses the suppression of all supersymmetry-breaking effects by $1/M_p^2$. (Here the quantities k_i denote dimensionless $O(1)$ numbers.) Although such a potential naturally gives a scalar mass of order $\mu \sim M_b^2/M_p \sim 1/r$, it also predicts a cubic coupling of order $g \sim (M_b/M_p)^2 \sim 1/(M_b r)^2$. Unfortunately, such a coupling cannot give a purely logarithmic potential, since it only contributes to higher order in $1/r$. This is what happens in the simplest models of supersymmetry breaking, as we now see.

B. Supergravity Models

Consider a model consisting of the nonchiral supergravity multiplet, as above, together with an additional matter multiplet consisting of a gauge potential and its superpartner: $\{A_M, \lambda\}$. With this model we take the dilaton, σ, to be the six-dimensional light scalar whose cubic couplings are of interest.

Rather than constructing its coupling to a brane, for calculational simplicity we instead break supersymmetry by artfully choosing the manifold on which we compactify the theory. In what follows we consider two such compactifications. We first compactify on a sphere, and generate in this way a cubic scalar coupling. Although this coupling is too small to generate a logarithmic potential, for reasons which are much as indicated above, it is nonetheless instructive. Our second compactification is on a torus, where we break supersymmetry completely using a Scherk-Schwarz mechanism [17]. (That is, we break supersymmetry by assigning different boundary conditions to different members of a supermultiplet.) With an eye to obtaining a positive radion potential, we require the dilaton to be antiperiodic about one of the cycles of the torus, and keep all of the rest of the fields periodic about both cycles. This compactification has the virtue of allowing many of the results of the previous section to be carried over in whole cloth.

The bosonic part of the supergravity action, coupled to these matter multiplets, is [14,15]

$$\begin{aligned}\frac{\mathcal{L}}{\sqrt{\mathcal{G}}} &= -\,\frac{1}{2\kappa^2}\,\mathcal{R} - \frac{1}{2}\,\partial_M\sigma\,\partial^M\sigma - \frac{1}{12}\,e^{2\kappa\sigma} G_{MNP}G^{MNP} \\ &\quad - \frac{1}{4}\,e^{\kappa\sigma}\,F_{MN}F^{MN} - \frac{2q^2}{\kappa^4}\,e^{-\kappa\sigma},\end{aligned} \tag{23}$$

where $G_{MNP} = 3\partial_{[M}B_{NP]} + 3\kappa F_{[MN}A_{P]}$ is the Kalb-Ramond field strength, F_{MN} is the usual abelian field strength for A_M, and all spinors carry a common charge, q, under the gauge group.[‡] Recall that in six dimensions the couplings have dimension $\kappa \propto \ell^2$ and $q \propto \ell$.

Compactification on a Sphere: The equations of motion obtained from this action admit a solution consisting of constant (and arbitrary) σ ($\partial_M\sigma = 0$), a gauge potential which is of the magnetic monopole form – with monopole number ± 1 – in two dimensions, and a metric which is the product between flat space and a two-sphere having radius r [16]:

$$\mathcal{G}_{MN} = \begin{pmatrix} \eta_{\mu\nu} & 0 \\ 0 & r^2\,\hat{g}_{mn} \end{pmatrix}. \tag{24}$$

$\hat{g}_{mn}$ is the metric on the (unit) two-sphere, and all other fields vanish: $G_{MNP} = \psi_M = \chi = \lambda = 0$. This compactification is known as the Salam-Sezgin model [16]. The flatness of the four-dimensional metric is only possible when the monopole number is $n = \pm 1$, and this may be understood from the fact that the solution leaves one four-dimensional supersymmetry unbroken only with this choice for the monopole number.

With this compactification a potential is generated for r and σ at tree level, due to the nonzero background values which are taken by the two-dimensional curvature scalar and electromagnetic field strength. The potential may be written as follows [18]:

$$V(r,\sigma) = \frac{2q^2 e^{-\kappa\sigma}}{\kappa^3 r^2}\left[1 - \frac{\kappa^2\,e^{\kappa\sigma}}{4q^2 r^2}\right]^2. \tag{25}$$

We see that the particular combination $X = e^{\kappa\sigma}/r^2$ appearing within the brackets has developed a potential, along whose minimum the potential vanishes. The combination of $Y = e^{-\kappa\sigma}/r^2$ appearing as a prefactor in eq. (25) is then seen to be a modulus of the compactification, parameterizing a flat direction along the bottom of this potential.

In principle, this has the form we seek. The field X is a six-dimensional scalar whose mass is naturally of order $1/r$. It also has a cubic self coupling, as measured by the third derivative of the potential in the X direction, evaluated at the minimum. Once the remaining

[‡]Since ψ_M and λ share the same chirality, while χ has opposite chirality, this theory as it stands has anomalies. We imagine these to be cancelled either by the addition of more matter fields or through a Green-Schwarz shift of B_{MN}, without affecting the rest of our analysis.

supersymmetry is broken, lifting the potential's degeneracy along the Y direction, one might hope to generate logarithmic terms in r along the lines described in the previous sections.

Unfortunately, the fly in the ointment is the size of the cubic coupling, which we see is of order $1/r^2$. Although this model nicely illustrates the existence of light scalars, and the generation of a potential for them, it does not furnish an example of a loop-generated logarithmic potential.

Toroidal Compactification: An alternative is to compactify on a torus, which allows us to use the results of the previous section's calculations for the radion potential. Toroidal compactifications are possible for the model in the case $q = 0$, in which case a classical solution is given by arbitrary, constant σ and a flat six-dimensional metric.

From the four-dimensional perspective, this vacuum solution preserves $N = 2$ supersymmetry and so no potential is generated for σ or r to any order in perturbation theory. In the absence of supersymmetry breaking, this can be seen explicitly at one loop as being due to the cancellation of the contributions of the various particles:

$$V_1^{\rm susy}(r) = V_\sigma^+(r) + V_{\rm rest}(r) = 0, \tag{26}$$

where $V_\sigma^+(r)$ denotes the contribution due to the σ field, and $V_{\rm rest}(r)$ is the one-loop contribution from all other particles. (The superscript '+' is meant as a reminder that eq. (26) is performed in a supersymmetric framework, with σ and all other fields periodic, say, about all cycles of the torus.)

Suppose we now break the supersymmetry, by assigning antiperiodic boundary conditions only to σ. Then the above cancellation no longer obtains, leaving the one-loop result:

$$\begin{aligned} V_1(r) &= V_\sigma^-(r) + V_{\rm rest}(r) = V_\sigma^-(r) - V_\sigma^+(r) \\ &= \frac{1}{2r^4}\int_0^\infty \frac{dx}{x^3}\Big[\vartheta_3(ix)\,\vartheta_3(ix) - \vartheta_2(ix)\,\vartheta_j(ix)\Big], \\ &= \begin{cases} +0.1643/r^4 & \text{if } j=3 \\ +0.2644/r^4 & \text{if } j=2 \end{cases} \end{aligned} \tag{27}$$

where $j = 3$ if σ is periodic about one cycle (and antiperiodic about the other) while $j = 2$ if σ is antiperiodic about both cycles.

In this model of supersymmetry breaking, the radiative-corrections computed in the previous section would directly apply, if there were only a nonzero cubic σ coupling. Although there is no potential for σ in the model as it stands, one can be generated by loop effects. Since our chosen method for supersymmetry breaking implies such a potential must vanish as $r \to \infty$, this should lead once more to cubic couplings which are proportional to powers of $1/r$.

Although we do not have a model which circumvents this difficulty, we do see reasons to be hopeful one could be constructed. The basic problem is to construct a model for which the scalar mass is suppressed by a symmetry, but where its cubic self-interactions are not similarly suppressed. One line of model building which this suggests is to tie the scalar to massless spin-one particles by having it lie in a gauge multiplet, since gauge boson masses can be forbidden by unbroken gauge symmetries without also precluding their having cubic self-interactions. (Scalar moduli in toroidal compactifications indeed typically do fall into four-dimensional gauge multiplets of $N = 2$ supersymmetry [19].) In this case one might plausibly hope that supersymmetry protects the scalar mass more strongly than it does the scalar's cubic couplings.

IV. PHENOMENOLOGICAL CONSEQUENCES AND CONSTRAINTS

What is generic about these models is their prediction of extremely light fundamental scalars which are gravitationally coupled. The two fields which are generic to the models we consider are the radion, r, and the light six-dimensional scalar (such as the dilaton, σ, in the supersymmetric examples just considered). We now turn to a discussion of the phenomenologically relevant properties of these scalars, and of the physical signatures which follow from these.

A. Masses and Couplings

The first question is the size of the scalar mass. We now compute this for the radion field, r, although similar considerations may also apply for the six-dimensional scalar, depending on the model. Although we have discussed the radion potential in some detail in previous sections, to determine its mass from this we must also compute the radion kinetic terms. Since r begins its life as part of the six-dimensional metric (*c.f.* eq. (24)) this kinetic energy may be read off (at tree level) from the six-dimensional Einstein-Hilbert action.

A straightforward dimensional reduction of this action using the metric

$$\mathcal{G}_{MN} = \begin{pmatrix} \hat g_{\mu\nu}(x) & 0 \\ 0 & \rho^2(x)\, h_{mn}(y) \end{pmatrix}, \tag{28}$$

with $\rho = r/\ell$, gives the result:

$$\mathcal{L}_{\rm kin} = -\frac{1}{2\ell^4}\int d^2y\,\sqrt{\mathcal{G}}\,\mathcal{R} \tag{29}$$

$$= -\frac{r^2}{2\ell^4}\sqrt{\hat g}\left[R(\hat g) - 2\left(\frac{\partial r}{r}\right)^2 + \frac{\ell^2 R(h)}{r^2}\right], \tag{30}$$

where we adopt the conventional normalization $\int d^2y\,\sqrt{h} = \ell^2$.

The Einstein-Hilbert term may be canonically normalized by rescaling $\hat{g}_{\mu\nu} = \rho^{-2} g_{\mu\nu}$, giving:

$$\mathcal{L}_{\rm kin} = -\frac{1}{2\ell^2}\,\sqrt{g}\left[R(g) + 4\left(\frac{\partial r}{r}\right)^2 + \frac{\ell^4 R(h)}{r^4}\right]. \quad (31)$$

From eq. (31) it is clear that the redefinition $r = \ell e^{\ell\xi/2}$ puts the kinetic term for ξ into canonical form. Adding this to an assumed logarithmic form for the potential, eq. (1), we have the four-dimensional radion-graviton dynamics relevant to cosmology given by:

$$\frac{\mathcal{L}}{\sqrt{g}} = -\,\frac{1}{2\ell^2} R(g) - \,\frac{1}{2}(\partial\xi)^2 - V(\xi), \quad (32)$$
$$V(\xi) = e^{-\lambda\ell\xi}\left[a_0 + 2a_1\ell\xi/2 + \cdots\right] + O\left[e^{-q\omega\ell\xi}\right],$$

with $\lambda = p/2 + 2$ (so $\lambda = 4$ if $V(r) \propto 1/r^4$).

We note here in passing that although an exponential potential, $V(\xi) = V_0\, e^{-\lambda\ell\xi}$, follows generically (at tree level) from the assumption of a power-law potential, $V(r) \sim 1/r^p$, the prediction $\lambda = (p+4)/2$ found above is not as robust. For instance, if the scalar potential were to mix ξ with another field – such as happened when r mixed with σ in the supergravity potential, eq. (25), of the previous section – then $\lambda = (p+4)/\sqrt{2}$.

To see how this comes about, consider an extreme example, where the potential at very low energies ($\sim 10^{-3}$ eV) is a function only of one combination of ξ and N other canonically-normalized fields, $\varphi_i, i = 1, ..., N$:

$$V \propto \exp[-\lambda_0(\xi + \varphi_1 + \varphi_2 + ... + \varphi_N)]. \quad (33)$$

In this case the canonically-normalized field which appears in the potential is $\hat{\xi} = (\xi + \varphi_1 + \cdots + \varphi_N)/\sqrt{N+1}$ and so in terms of this variable the argument of the exponential is: $-\lambda_0\sqrt{N+1}\;\hat{\xi}$, leading to the prediction $\lambda = \lambda_0\sqrt{N+1}$. The supergravity case has precisely this form with $\lambda_0 = \frac{1}{2}(p+4)$ and $N = 1$.

Regardless of the value found for λ, the mass which results from these manipulations is *extremely* small, being of order

$$m \sim \frac{1}{M_p r^2} \sim 10^{-33}\text{eV} \quad (34)$$

if $1/r \sim 10^{-3}$ eV, with the decisive suppression by M_p arising because of the radion's kinetic term sharing a common origin with the four-dimensional Einstein-Hilbert action.

Before turning to the strong experimental constraints which any such scalar must satisfy, we make a brief aside to check whether such a small scalar mass is technically natural. That is, we ask if the small mass we have found is an artifact of the tree approximation, or if it is stable under quantum corrections and renormalization.

B. Naturalness

We now argue that masses as incredibly small as those of eq. (34) can be stable under radiative correction, in models such as were considered in the previous sections [23]. This remarkable stability may be seen most easily by integrating out physics at successive scales, and asking how large the contributions to the scalar mass must be as these scales are integrated out.

There are two main scales on which to focus. First we integrate out all physics in the six-dimensional theory between the microscopic scale $M_b = 1/\ell$ and the extra-dimensional radius r. In the models of interest we have supersymmetry in the bulk space, with supersymmetry broken at scale ℓ on the branes on which we live. Since all bulk-brane couplings are gravitational in strength, we saw that supersymmetry-breaking interactions of the four-dimensional gravity multiplet are suppressed by powers of $1/M_p \propto 1/r$. On dimensional grounds, any supersymmetry-breaking mass splittings within the four-dimensional gravity multiplet are at most of order $M_b^2/M_p \sim 1/r$. This is indeed the typical size expected for the masses of the Kaluza-Klein (KK) states associated with the six-dimensional metric multiplet.

We can see how this estimate works for the lowest KK states (like the radion) by using our calculation of the radion potential in earlier sections. There we found $V(r) = O(1/r^4)$, and so its second derivative is $d^2V/dr^2 = O(1/r^6)$. We found this form for V to be stable under radiative corrections, with generic corrections introducing higher powers of $1/r$, and the marginal cubic interaction introducing logarithms of r. If the kinetic term for the field r also had its scale set by r, we would have found masses of order $1/r$, as expected by the dimensional argument of the previous paragraph.

Instead, because the tree-level kinetic terms are $O(M_p^2)$ we find the much smaller mass, $O(1/M_p r^2)$. But these tree-level kinetic terms dominate any radiative corrections to the kinetic terms, which are suppressed relative to the tree contribution by powers of $1/r$ (or logarithms – see below). So we expect our mass estimate to be stable under the renormalization through the six-dimensional theory between the scales ℓ and r.

In the effective four-dimensional theory for distances larger than r, the ultraviolet cutoff is given by $\Lambda \sim 1/r$, and all couplings of the scalar within the effective theory are suppressed by at least one power of $1/M_p$. Although supersymmetry-breaking effects are not particularly small in this effective theory, corrections to scalar masses within the theory are of order $\Lambda^2/M_p \sim 1/(M_p r^2)$, making them the same size as the scalar masses themselves. We therefore expect these masses to be not changed (in order of magnitude) under these lower-energy radiative corrections.

C. Kinetic Term Corrections

Although they do not change the order of magnitude of the mass inferred, radiative corrections to the kinetic terms do have an important impact on our later confrontation with experimental bounds, and so we pause to consider them briefly here.

We have seen that $\alpha(r)$ corrections can modify the potential at large r in an important way, by adding logarithmic corrections in r. The same is true for the radion kinetic terms, which are also dominated for $r \gg \ell$ by logarithmic corrections arising due to powers of $\alpha(r)$. Including these corrections, the relevant kinetic terms in the effective theory at large scales have the form:

$$\mathcal{L}_{\rm kin} = -\frac{1}{2\ell^2} \sqrt{g} A^2[\alpha] \left[R(g) + 4B^2[\alpha] \left(\frac{\partial r}{r} \right)^2 \right], \quad (35)$$

where both A and B have the series expansion $1 + O(\alpha)$.

In general, the function B changes the field redefinition, $\xi(r)$, which is required to canonically normalize the radion kinetic terms. For general A and B we have

$$\xi = \frac{2}{\ell} \int_\ell^r F[\alpha(r')] \, \frac{dr'}{r'}, \quad (36)$$

where $F^2 = B^2 + \frac{3}{2} (A'/A)^2 \beta^2$, $A' = dA/d\alpha$ and $\beta = r d\alpha/dr$.

A and B are independent of r only to next-to-lowest order, where $A \approx 1 + a\alpha_0$ and $B \approx 1 + b\alpha_0$, and so to this order ξ is still logarithmically related to r, but we have $\lambda \approx (\frac{1}{2}p + 2)[1 - b\alpha_0]$. Beyond this order ξ need not be strictly logarithmic in r, and so the functional form of the potential $V(\xi)$ changes in a more complicated way. We shall see that this has important implications for the cosmology of the radion field.

D. Experimental Constraints

We see there are three important mass scales for the bulk sector of these models: the microscopic scale, $1/\ell$; the Kaluza-Klein mass scale, $1/r$; and the radion mass scale, $1/(M_p r^2)$. These imply potentially interesting modifications of gravity on a variety of scales. For instance, the intermediate-scale scenario — $1/\ell \sim 10^{10}$ GeV and $1/r \sim 1$ TeV — puts $1/(M_p r^2) \sim 10^{-3}$ eV into the millimetre range. Alternatively, for large extra dimensions $1/r \sim 10^{-3}$ eV and $1/M_p r^2 \sim 10^{-33}$ eV.

Millimetre Scales

As the above examples show, very different choices for ℓ and r imply modifications to gravity at scales of order 10^{-3} eV. The implications for experiments probing submillimetre range forces can differ dramatically depending on the nature of the microscopic physics. In the most extreme case $1/r \sim 10^{-3}$ eV and any observed modifications to Newton's Law on these scales would signal the transition to six-dimensional gravitational physics. Searches for such deviations are now starting to probe forces within this interesting range [24].

The exact kinds of signals experiments searching for these deviations should expect to see depend on the precise nature of the couplings of the relevant states to ordinary matter. Unfortunately these are difficult to cleanly predict, since they depend on the scenario involved. Furthermore, if the modifications are due to the onset of 6-dimensional physics, then predictions are also hampered by the large size of $\alpha(r)$ whose growth at low energies underlies our mechanism for generating large radii. One generically expects deviations from the equivalence principle, and from the $1/r$ falloff of the gravitational potential, at scales smaller than a millimetre.

Very Long-Range Forces

Since the masses of some of the lightest states, like the radion, can be incredibly small, $\sim 10^{-33}$ eV, their couplings are very strongly constrained. Such small masses make the radion's Compton wavelength, $1/m$, of order 10^{26} m, permitting them to mediate extremely long-ranged forces. We will argue here that the properties of the scalar predicted by our mechanism for radius stabilization can evade these bounds, and do so in an interesting way.

If the tree-level action of eqs. (29) and (31) were the whole story, our model would be ruled out. Since ordinary matter, trapped as it is on a four-dimensional brane within the six dimensions of the bulk, couples to the radion only indirectly through $\hat{g}_{\mu\nu}$, these expressions, with the field redefinition $\Phi = r^2$ show that the radion behaves precisely as a Brans-Dicke scalar, with coupling parameter $\omega = -\frac{1}{2}$. Although Brans-Dicke scalars can be phenomenologically acceptable, even if they are massless, solar-system tests require their couplings must be strongly suppressed relative to gravity, with current constraints requiring $\omega \gtrsim 3,000$ [24].

The story is much more interesting once the radiative corrections due to $\alpha(r)$ are included, since these imply ω becomes a function of r. For weak coupling, for instance, $\omega(\Phi) = -\frac{1}{2} + \omega_1 \alpha_0 \log \Phi + \cdots$.

The recognition that ω is a function of Φ is crucial when comparing with experiments, since it implies in particular that ω is a function of time as the universe evolves cosmologically. Since the best limits only apply at the current epoch, the constraints are satisfied so long as $\omega(t)$ approaches sufficiently large values sufficiently quickly as the universe evolves towards the present. Better yet, the evolution of $\omega(\Phi)$ in general scalar-tensor theories has been found to generically be attracted towards large ω

during cosmological evolution [25], indicating that current bounds can be generically satisfied without making unreasonable assumptions about the cosmological initial conditions.

As we report in more detail in a companion publication [26], we have applied the analysis of ref. [25] to realistic cosmological evolution, using the functions $\omega(r)$ and $V(r)$ which are plausibly obtained within our scenario. We find that current constraints on post-Newtonian gravity and constraints from cosmology can all be satisfied, and that it is also easy to arrange the energy density of the scalar field to be currently starting to dominate the energy density of the universe, making the radion a natural microscopic realization of the 'Planck scale quintessence' described in [8].

V. SUMMARY

Our purpose has been to propose a mechanism for naturally generating exponentially large radii, within a plausible scenario of extra-dimensional physics. Besides making the usual assumption that there is no large microscopic cosmological constant, our mechanism requires the following three ingredients:

1. The universe must pass through a phase during which there are effectively six large dimensions;
2. A very light – mass $m \lesssim O(1/r)$ – spinless particle must be present within the six-dimensional effective theory;
3. The light scalar must have a cubic self interaction in the effective six-dimensional theory, whose coupling g_0 is not itself already suppressed by powers of $1/r$.

Under these circumstances we have shown that the renormalization of the one marginal six-dimensional coupling, g_0, generically generates a potential energy which depends logarithmically on the radius, r, of the 2 extra dimensions.

Besides generically having a runaway minimum, with $r \to \infty$, this potential also has a minimum with $r = \ell\ \exp[+(4\pi)^3/g_0^2]$, where ℓ is the length scale of the microscopic six-dimensional theory. The existence of this minimum depends delicately on the signs of the renormalization group beta function for the coupling g, which cannot be adjusted. Within the context where the radion potential is generated as a Casimir energy (such as due to the light six-dimensional scalar) we have shown that the beta function signs are the ones required to produce the exponentially large radius.

In order to determine how natural our above three requirements are, we explored several kinds of six-dimensional models explicitly. We have found that supersymmetry can naturally assure items 1 and 2 in the list above, but assuring item 3 is more difficult. In the models explored, supersymmetry both protects the scalar mass *and* its cubic coupling to be of order $1/r$, violating item 3. Unfortunately, it is difficult to exhaustively explore the potentials which can be generated for such scalars given the present poor understanding of how supersymmetry breaks.

The same features of the radion potential which allow it to have naturally large extrema, also suggest its interpretation as the source of quintessence, accounting for the present-day cosmological constant. They similarly allow the radion to evade all experimental constraints, despite the extremely small masses which are possible: $m \sim 10^{-33}$ eV. Both the evasion of the bounds and the cosmological interpretation as quintessence rely crucially on the existence of the same radiative corrections which generate the exponentially large radii, since these corrections convert the radion from an ordinary Brans-Dicke scalar to a more general scalar-tensor theory for which the couplings to matter evolve cosmologically to small values.

VI. ACKNOWLEDGEMENTS

We would like to acknowledge J. Maldacena, N. Seiberg, S. Shenker and X. Tata for useful suggestions, and the Aspen Centre for Physics for providing the pleasant environs in which this work started. CB's research is partially funded by NSERC (Canada), FCAR (Québec) and the Ambrose Monell Foundation. AA and CS are supported by DOE grant DE-FG03-91ER40674 and U.C. Davis.

VII. APPENDIX A: THE ONE LOOP CASIMIR ENERGY

In this appendix we provide expressions for the one-loop Casimir energy (as a function of radius) of a six-dimensional scalar (having mass μ) on the product of four-dimensional Minkowski space with a torus. We compute this for scalars which are periodic or antiperiodic about the torus' two nontrivial cycles. With these boundary conditions the momenta of the scalar in the toroidal directions take values $(n + a, m + b)$ in units of $2\pi/r$, where the constants a and b are zero if the scalar is periodic about these cycles and are $\frac{1}{2}$ if it is antiperiodic.

The one loop vacuum energy for such a scalar, is given (for Euclidean momenta) by

$$\begin{aligned}\Lambda_1 &= \frac{1}{2r^2} \sum_{mn=-\infty}^{\infty} \int \frac{d^4k}{(2\pi)^4}\ \log\Big(k^2 + \mu^2_{mn}\Big), \qquad (37)\\ &= -\frac{1}{2r^2} \int_0^\infty \frac{dy}{y} \int \frac{d^4k}{(2\pi)^4} \sum_{mn=-\infty}^{\infty} \exp\Big[-y(k^2 + \mu^2_{mn})\Big],\end{aligned}$$

where

$$\mu_{mn}^2 = \mu^2 + \left(\frac{2\pi}{r}\right)^2 \Big[(m+a)^2 + (n+b)^2\Big]. \qquad (38)$$

In the second of eqs. (37) the sums and the integrals over k can be performed explicitly in terms of Jacobi theta-functions, using

$$\begin{aligned}\int \frac{dk}{2\pi} \; \exp\Big[-yk^2\Big] &= \frac{1}{\sqrt{4\pi y}}, \\ \sum_{n=-\infty}^{\infty} \exp\left[-\frac{4\pi^2 y}{r^2}\; n^2\right] &= \vartheta_3\,(\tau)\,, \qquad (39)\\ \sum_{n=-\infty}^{\infty} \exp\left[-\frac{4\pi^2 y}{r^2}\;\left(n+\frac{1}{2}\right)^2\right] &= \vartheta_2\,(\tau)\,,\end{aligned}$$

with $\tau = 4\pi i y/r^2$. Using these in eq. (37), and integrating the result over the torus to obtain a four-dimensional effective potential

$$V(r) = \int d^2z \; \Lambda = r^2\,\Lambda, \qquad (40)$$

leads to

$$V_1(r) = -\,\frac{1}{2r^4}\int_0^{\infty} \frac{dx}{x^3}\; e^{-\beta x}\;\vartheta_j(ix)\,\vartheta_k(ix), \qquad (41)$$

where $\beta = \mu^2 r^2/(4\pi)$ and $j,k = 2,3$ according to the boundary conditions which are appropriate.

Useful asymptotic forms for the theta functions are:

$$\begin{aligned}\vartheta_2(ix) &= 2\,e^{-\pi x/4} + \cdots \qquad &&\text{as } x\to\infty\\ \vartheta_3(ix) &= 1 + 2\,e^{-\pi x} + \cdots \qquad &&\text{as } x\to\infty \qquad (42)\\ \vartheta_2(ix) &= \frac{1}{\sqrt{x}}\Big[1 - 2\,e^{-\pi/x} + \cdots\Big] \qquad &&\text{as } x\to 0\\ \vartheta_3(ix) &= \frac{1}{\sqrt{x}}\Big[1 + 2\,e^{-\pi/x} + \cdots\Big] \qquad &&\text{as } x\to 0\end{aligned}$$

and these show that the one-loop vacuum energy converges in the infrared ($x\to\infty$) even if μ vanishes. For antiperiodic boundary conditions this convergence is exponential, reflecting the absence of exactly massless four-dimensional modes in this case.

On the other hand, the vacuum energy diverges in the ultraviolet ($x\to 0$), but this divergence is independent of r and so may be absorbed into a renormalization of the six-dimensional cosmological constant. The finite, r-dependent result is obtained by subtracting the result for $r\to\infty$, giving eq. (8).

VIII. APPENDIX B: THE TWO-LOOP CASIMIR ENERGY

In this appendix we compute the contribution to the Casimir energy coming from the two-loop graph of Fig. (2), plus the graph in which a wavefunction and mass renormalization counterterm is inserted into the one-loop Casimir energy. We show that the result converges, as expected on general grounds, in the infrared and ultraviolet, and evaluate the constant $c_4^{(2)}$ which controls its overall size.

Our starting point is the contribution to the four-dimensional energy density coming from the evaluation of Fig. (2):

$$\begin{aligned}V_2^{\rm Fg2}(r) &= -\,\frac{g^2}{12\,r^2}\sum_{jkln}\int \frac{d^4p}{(2\pi)^4}\,\frac{d^4q}{(2\pi)^4} \qquad (43)\\ &\quad\times \frac{1}{(p^2+\mu_{jk}^2)(q^2+\mu_{ln}^2)((p+q)^2+\mu_{j+l,k+n}^2)}.\end{aligned}$$

As before, $\mu_{nl}^2 = \mu^2 + (2\pi/r)^2[(n+a)^2+(l+b)^2]$, where $a,b = 0,\frac{1}{2}$ reflect the scalar boundary conditions and n,l are integers.

Use of the identity $1/X = \int_0^\infty ds\; e^{-sX}$ for each propagator permits the performance of the sums and integrals. Three results are required beyond those used in Appendix A:

$$\begin{aligned}\int \frac{dp}{2\pi}\; e^{-ap^2-bp} &= \frac{e^{b^2/4a}}{\sqrt{4\pi a}}, \qquad (44)\\ \sum_k e^{-ak^2-bk} &= \vartheta_3\left[\frac{ib}{2}, e^{-a}\right],\\ \sum_k e^{-a(k+\frac{1}{2})^2-b(k+\frac{1}{2})} &= \vartheta_2\left[\frac{ib}{2}, e^{-a}\right],\end{aligned}$$

where $\vartheta_k(z,q)$ – defined in Ref. [11] – has two complex arguments, and is related to the theta-function with one argument by $\vartheta_k(\tau) = \vartheta_k(0,q)$ with $q = e^{i\pi\tau}$.

Finally, we require the sum

$$\sum_{njkl} e^{-s\mu_{nj}^2 - t\mu_{kl}^2 - u\mu_{n+k,j+l}^2} = e^{-\beta(s+t+u)}\; S_c^2, \qquad (45)$$

where $\beta = \mu^2 r^2/(4\pi)$ and we define the function S_c, with $c = 2,3$ by:

$$S_c = \sum_n e^{-\pi(u+s)(n+a_c)^2}\,\vartheta_c\left[i\pi(n+a_c)u, e^{-\pi(t+u)}\right]. \qquad (46)$$

Here $c=3$ and $a_3 = 0$ for periodic boundary conditions, while $c=2$ and $a_2 = \frac{1}{2}$ for antiperiodic conditions.

Combining everything gives the following contribution to $V(r)$:

$$V_2^{\rm Fg2} = -\,\frac{g^2}{12(4\pi)^3 r^4}\int_0^\infty \frac{ds\,dt\,du}{(st+su+ut)^2}\; e^{-\beta(s+t+u)}\; S_c^2. \qquad (47)$$

This expression converges in the infrared, even if $\mu\to 0$. It has several sources of diverence in the ultraviolet.

The divergence when all three variables, s, t and u, vanish is removed by subtracting the result for $r \to \infty$, implying that it is removed by renormalizing the six-dimensional cosmological constant. Using the asymptotic form

$$\vartheta_c\left(z, e^{-\pi x}\right) = \frac{e^{-z^2/(\pi x)}}{\sqrt{x}} \left[1 + O\left(e^{-\pi/x}\right)\right], \tag{48}$$

for $x \to 0$, one finds for $u + t \ll 1$:

$$S_c = \frac{1}{\sqrt{u+t}}\, \vartheta_c \left[\frac{i(st+su+ut)}{u+t}\right] \left[1 + O\left(e^{-\pi/(t+u)}\right)\right]. \tag{49}$$

The result, after subtracting the large-r limit is therefore obtained from eq. (47) by replacing S_c^2 by

$$\hat{S}_c^2 := S_c^2 - \frac{1}{st+su+tu}. \tag{50}$$

Although this subtraction renders finite the limit where s, t, u all vanish with fixed nonzero ratios, it does not cure the ultraviolet divergence when two of the variables s, t, u vanish with the third held fixed. This divergence cancels with the result obtained when the counterterms for wavefunction and mass renormalization are inserted into the one-loop Casimir energy.

To see how this works, notice that an evaluation of the one-loop scalar self-energy for the torus gives:

$$\begin{aligned}\Pi(p^2) &= \frac{g^2}{2r^2}\int_0^\infty dt\, du \\ &\qquad \times \sum_{ln} \int \frac{d^4k}{(2\pi)^4}\, e^{-t(k^2+\mu_{ln}^2)-u((k-p)^2+\mu_{ln}^2)} \\ &= \frac{g^2}{2r^2(4\pi)^2}\int_0^\infty \frac{du\, dt}{(u+t)^2}\, e^{-\beta(u+t)} \\ &\qquad \times e^{-\left(\frac{ut}{u+t}\right)\frac{p^2r^2}{4\pi}}\, \vartheta_j[i(u+t)]\vartheta_k[i(u+t)].\end{aligned} \tag{51}$$

For an on-shell renormalization scheme this gives the mass and wavefunction counterterms as:

$$\begin{aligned}\delta Z &= \frac{g^2}{2(4\pi)^3}\int_0^\infty du\, dt\, \frac{ut}{(u+t)^3}\, e^{-\beta(u+t)} \\ &\qquad \times\vartheta_j[i(u+t)]\vartheta_k[i(u+t)] \\ \mu^2\delta Z + \delta\mu^2 &= -\frac{g^2}{2r^2(4\pi)^2}\int_0^\infty \frac{du\, dt}{(u+t)^2}\, e^{-\beta(u+t)} \\ &\qquad \times\vartheta_j[i(u+t)]\vartheta_k[i(u+t)].\end{aligned} \tag{52}$$

Denoting $\xi(p^2) = \delta Z(p^2+\mu^2) + \delta\mu^2$, the insertion of these counterterms into the one-loop Casimir energy gives

$$\begin{aligned}V_2^{ct} &= -\frac{1}{r^2}\int_0^\infty ds \sum_{ln}\int \frac{d^4p}{(2\pi)^4}\, \xi(p^2) e^{-s(p^2+\mu_{ln}^2)} \\ &= \frac{g^2}{6(4\pi)^3 r^4}\int_0^\infty \frac{ds\, dt\, du}{(s+t+u)^2}\, e^{-\beta(s+t+u)} T_{jk}(s,t,u),\end{aligned} \tag{53}$$

where we have symmetrized the result with respect to permutations of s, t, u and the function T_{jk} is defined by

$$\begin{aligned}T_{jk} := \left[1 - \frac{2ut}{s(u+t)}\right] \frac{\vartheta_j(is)\vartheta_k(is)}{s^2(t+u)^3} \\ +(\text{cyclic permutations of } s,t,u).\end{aligned} \tag{54}$$

The two-loop contribution to the radion potential is obtained by summing $V_2^{ct}(r)$ with the result, eq. (47), from Fig. (2), and then subtracting the result with $r \to \infty$ from the sum. It is both ultraviolet and infrared finite, and when evaluated at $\mu = 0$ gives expression (17) of the text.

[1] S. Perlmutter et al., Ap. J. **483** 565 (1997) (astro-ph/9712212); A.G. Riess *et al*, Ast. J. **116** 1009 (1997) (astro-ph/9805201);N. Bahcall, *et al. Science* **284**, 1481, (1999).

[2] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. **B429** (1998) 263 (hep-ph/9803315); Phys. Rev. **D59** (1999) 086004 (hep-ph/9807344); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. **B436** (1998) 257 (hep-ph/9804398); P. Horava and E. Witten, Nucl. Phys. **B475** (1996) 94 (hep-th/9603142); Nucl. Phys. **B460** (1996) 506 (hep-th/9510209); E. Witten, Nucl. Phys. **B471** (1996) 135 (hep-th/9602070); J. Lykken, Phys. Rev. **D54** (1996) 3693 (hep-th/9603133); I. Antoniadis, Phys. Lett. **B246** (1990) 377.

[3] L. Randall, R. Sundrum, *Phys. Rev. Lett.* **83** (1999) 3370 (hep-ph/9905221), *Phys. Rev. Lett.* **83** (1999) 4690 (hep-th/9906064).

[4] S. Cullen and M. Perelstein, *Phys. Rev. Lett.* **83** (1999) 268 (hep-ph/9903422); C. Hanhart, D.R. Phillips, S. Reddy, M.J. Savage, *Nucl. Phys.* **B595** (2001) 335 (nucl-th/0007016).

[5] D. Atwood, C.P. Burgess, E. Filotas, F. Leblond, D. London and I. Maksymyk, *Phys. Rev.* **D63** (2001) 025007 (hep-ph/0007178).

[6] J.-W. Chen, M.A. Luty and E. Pontón, (hep-th/0003067).

[7] N. Arkani-Hamed, L. Hall, D. Smith and N. Weiner, Phys.Rev. **D62** 105002 (2000) (hep-ph/9912453).

[8] A. Albrecht and C. Skordis, *Phys. Rev. Lett.* **84** 2076 (2000) (astro-ph/9908085).

[9] J. Weller, in the proceedings of PASCOS 99, J. Gunion, Ed. (astro-ph/0004096).

[10] For a recent discussion of radius stabilization using Casimir energies see E. Pontón and E. Poppitz, preprint (hep-th/0105021).

[11] Whittaker and Watson,*A course in modern analysis* Cambridge Univ. Press.

[12] C.P. Burgess, L.E. Ibañez and F. Quevedo, *Phys. Lett.* **B447** (1999) 257; K. Benakli, *Phys. Rev.* **D60** (1999) 104002 (hep-ph/9809502).

e-Print Archive: hep-ph/9810535

[13] J. Long, H. Chan, J. Price, , Nucl.Phys. **B539** (1999) 23-34 (hep-ph/9805217). For future experiments see J. Long, A. Churnside, J. Price, (hep-ph/0009062).

[14] N. Marcus and J.H. Schwarz, *Phys. Lett.* **115B** (1982) 111.

[15] H. Nishino and E. Sezgin, *Phys. Lett.* **144B** (1984) 187.

[16] A. Salam and E. Sezgin, *Phys. Lett.* **147B** (1984) 47.

[17] J. Scherk and J.H. Schwarz, *Phys. Lett.* **B82** (1979) 60.

[18] J.J. Halliwell, *Nucl. Phys.* **B286** (1987) 729.

[19] A. Ceresole, R. D'Auria, S. Ferrara and A. Van Proeyen, (hep-th/9412200); B. de Wit, V. Kaplunovsky, J. Louis and D. Lüst, Nucl.Phys. **B451** 53 (1995) (hep-th/9504006); G. Lopes Cardoso, G. Curio and D. Lüst, Nucl.Phys. **B491** 147 (1997) (hep-th/9608154).

[20] J.J. Halliwell, *Phys. Lett.* **185B** (1987) 341; J. Barrow, *Phys. Lett.* **187B** (1987) 12; E. Copeland, A. Liddle and D. Wands, *Ann. NY Acad. Sci.* **688** (1993) 647; C. Wetterich, *Astron. and Astrophy.* **301** (1995) 321; B. Ratra and P. Peebles, *Phys. Rev.* **D37** (1988) 3406; T. Barreiro, B. de Carlos, E.J. Copeland, *Phys. Rev.* **D58** (1998) 083513; E. Copeland, A. Liddle and D. Wands, *Phys. Rev.* **D57** (1998) 4686.

[21] P. Ferreira and M. Joyce, *Phys. Rev.* **D58** (1998) 023503.

[22] C.P.Burgess, R. Myers and F. Quevedo, *Phys. Lett.* **495B** (2000) 384 (hep-th/9911164).

[23] For other recent discussions of very light scalar masses see: G. Dvali, G. Gabadadze and M. Porrati, *Phys. Lett.* **B485** (2000) 208 (hep-th/0005016).

[24] For a recent summary of experimental bounds on deviations from General Relativity, see C.M. Will, Lecture notes from the 1998 SLAC Summer Institute on Particle Physics (gr-qc/9811036); C.M. Will(gr-qc/0103036).

[25] T. Damour and K. Nordtvedt, *Phys. Rev.* **48** (1993) 3436.

[26] A. Albrecht, C.P. Burgess, F. Ravndal and C. Skordis, in preparation.